

# Lighthouses with two lights: burst oscillations from the accretion-powered millisecond pulsars

Anna L. Watts

*Astronomical Institute “Anton Pannekoek”, University of Amsterdam, Kruislaan 403,  
1098 SJ Amsterdam, the Netherlands; A.L.Watts@uva.nl*

**Abstract.** The key contribution of the discovery of nuclear-powered pulsations from the accretion-powered millisecond pulsars (AMPs) has been the establishment of burst oscillation frequency as a reliable proxy for stellar spin rate. This has doubled the sample of rapidly-rotating accreting neutron stars and revealed the unexpected absence of any stars rotating near the break-up limit. The resulting ‘braking problem’ is now a major concern for theorists, particularly given the possible role of gravitational wave emission in limiting spin. This, however, is not the only area where burst oscillations from the AMPs are having an impact. Burst oscillation timing is developing into a promising technique for verifying the level of spin variability in the AMPs (a topic of considerable debate). These sources also provide unique input to our efforts to understand the still-elusive burst oscillation mechanism. This is because they are the only stars where we can reliably gauge the role of uneven fuel deposition and, of course, the magnetic field.

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## 1. INTRODUCTION

‘History’, as George Orwell once noted, ‘is written by the winners’ [1] - or, in this case, by the workshop hosts. When we gathered in Amsterdam in April 2008, it was ostensibly to celebrate ten years since the discovery of the first Accreting Millisecond X-ray Pulsar (AMXP). We were, however, a full two years too late. For the first AMXP was not SAX J1808.4-3658 [2], but rather the far less well-known 4U 1728-34 [3]. How on earth, you might ask, could such a slip go unnoticed? The trick, of course, lies in the terminology. Most astronomers (the author included) tend to think of the AMXPs as comprising only the *accretion-powered* millisecond pulsars (AMPs), forgetting the equally large class of *nuclear-powered* millisecond pulsars (NMPs) - the burst oscillation sources.

Most of this volume focuses on the AMPs, where persistent pulsations are generated as accreting material is channeled by the magnetic field onto magnetic polar caps that are offset from the rotational poles. The NMPs, by contrast, show pulsations during Type I X-ray bursts (thermonuclear explosions on the stellar surface caused by rapid unstable burning of accreted material). The cause of the brightness asymmetry in the NMPs remains an open question [4, 5], and to do full justice to NMP phenomenology would merit a much longer discussion. In this article, however, I will focus on the small set of NMPs that are also AMPs. These rare objects provide a unique insight into many current problems in neutron star astrophysics because, as suggested by my title, they are lighthouses with two different light sources. The accretion-powered pulsations tell us how the material arrives on the stellar surface, while the nuclear-powered pulsations tell

us what happens once it gets there.

Section 2 provides a brief overview of the relevant observational results. The bulk of the review focuses, however, on the astrophysical questions where these sources have made or are making a major contribution to our understanding. These include the spin distribution of AMXPs, torque modeling, and the burst oscillation mechanism.

## 2. OBSERVATIONAL SUMMARY

Type I X-ray bursts have been observed from two of the seven persistent<sup>1</sup>, and all three of the intermittent, AMPs. The AMPs for which no bursts have been detected are the four ultra-compact systems (XTE J1751-305, XTE J1807-294, XTE J0929-314 and SWIFT J1756.9-2508) and IGR J00291+5934. The latter has a similar orbital period to the bursting AMPs, so is a good candidate for burst detection during its next outburst<sup>2</sup>.

The first AMP to be detected as an NMP was SAX J1808.4-3658 [7]. The burst oscillations exhibit frequency drifts of a few Hz in the rising phase of the brightest bursts, settling down to a frequency that is within 0.1 Hz of the spin frequency in the burst tail. This result was followed by the discovery of burst oscillations in XTE J1814-338 [8]. In this source, the nuclear-powered pulsation frequency is extremely stable, and equal to the spin frequency inferred from the accretion-powered pulsations [9].

Of the three intermittent AMPs only Aql X-1 has burst oscillations. Indeed this source was discovered to be an NMP [10] long before its detection as an intermittent AMP [11]. The frequency inferred from the accretion-powered pulsations is offset by a small amount ( $< 1$  Hz) from the asymptotic frequency of the burst oscillations (as in many other sources, the burst oscillations from Aql X-1 drift in frequency during a burst but asymptote to a frequency that remains stable from burst to burst [12]). Burst oscillations have not been detected in any of the bursts from HETE J1900.1-2455 or SAX J1748.9-2021<sup>3</sup> that have been recorded with high time resolution instruments [5].

## 3. NEUTRON STAR SPIN

### 3.1. Spin distribution and the paucity of rapid rotators

Identifying the spin distribution of the various classes of neutron star has been a major research goal for many years. Spin rates are important to our understanding of stellar and binary evolution: finding rapidly-rotating accreting neutron stars has been critical, for example, to confirming the recycling scenario for the formation of the millisecond radio

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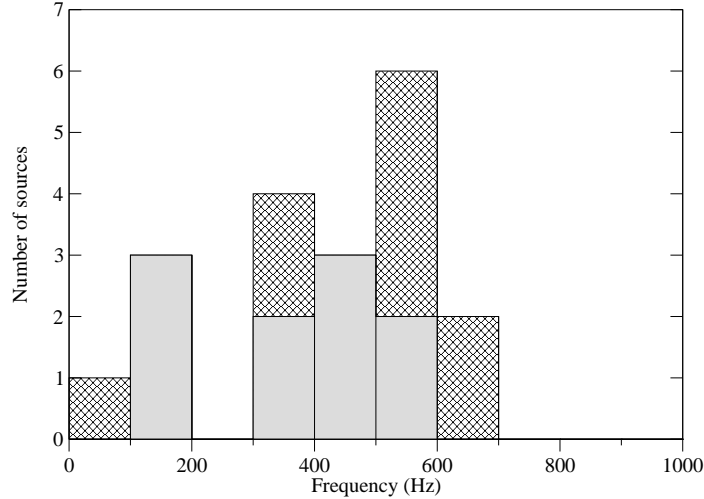
<sup>1</sup> Here I am referring to the persistence of accretion-powered pulsations throughout an accretion episode. The intermittent pulsators, by contrast, have detectable accretion-powered pulsations only sporadically during accretion episodes. Confusingly, the term persistent is also used for systems that accrete at detectable levels (measured via X-ray emission) all the time. Transients, on the other hand, accrete only occasionally, with alternating periods of outburst (not to be confused with bursts!) and quiescence.

<sup>2</sup> A new outburst started just as this article was being completed [6].

<sup>3</sup> The statistical significance of the burst oscillation claimed by [13] is much lower than quoted [14].

pulsars [15]. Maximum spin rates are also important to relativistic nuclear astrophysics, since they place firm constraints on equations of state [16].

Identifying the most rapidly-rotating X-ray pulsars, however, has proved to be far more difficult than for the radio pulsars. The sources are far less numerous, and the vast majority of neutron stars in Low Mass X-ray Binaries (LMXBs) do not have accretion-powered pulsations. The discovery of the NMPs promised a substantial enlargement of the sample of sources with measured spins, if it could be proven that burst oscillation frequency was the spin frequency. Frequency drifts during bursts [12], and the failure to identify the mechanism responsible for the nuclear-powered pulsations, had led to some caution - although repeated measurements of the same burst oscillation frequency for a given source [12], and the presence of oscillations at the same frequencies in both Type I bursts and superbursts [17], certainly supported the hypothesis. The eventual detection of burst oscillations from the AMPs, at the spin frequency, now seems to have resolved this question. The correspondence between the two frequencies is not exact for SAX J1808.4-3658 and Aql X-1 (which have drifting burst oscillation frequencies), but identification to within a few Hz is close enough for most practical purposes.



**FIGURE 1.** The distribution of measured spin rates inferred from the AMPs (grey) and NMPs (hatched). SAX J1808.4-3658, XTE J1814-338 and Aql X-1 are included in the AMP sample, rather than the NMP sample. We only include NMPs where the same burst oscillation frequency has been seen in more than one burst, since these are the statistically secure detections (see [18] for further discussion of this issue). We do not include any source with spin below 40 Hz in this Figure.

The main consequence has been a doubling of the number of rapidly-rotating accreting neutron stars with a measured spin rate (Figure 1). This has brought to light a new and interesting problem. Simple estimates of accretion-induced spin-up over the lifetime of an LMXB suggest that there should be a population of neutron stars with spin frequencies above 1 kHz [19] (and such rapid spin rates are permitted by all modern equations of state [16]). However the fastest AMXP spins at 620 Hz [20], while the fastest radio pulsar spins at 716 Hz [21]. If the evolutionary estimates are sound, there is a requirement for a braking mechanism to halt the spin-up. Magnetic braking (due to interaction between the stellar field and the accretion disk) is one possibility [22, 23, 24]. Another, which has generated a lot of excitement, is the emission of gravitational waves

[25, 26, 27]. This could make AMXPs promising sources for future gravitational wave detectors, although ironically this is one application where knowing the spin to a very high degree of precision would be important [18].

It is of course worth remembering that we still do not understand the mechanism responsible for burst oscillations. The burst oscillation properties of SAX J1808.4-3658 and XTE J1814-338 are rather unusual compared to the rest of the NMPs (Section 4), and it is still possible that the cause of the nuclear-powered pulsations may differ for these sources. The results from Aql X-1 are reassuring in this regard, but to have full confidence in the use of burst oscillations as a spin proxy the mechanism must be uncovered.

### **3.2. Spin variation and torques in neutron stars**

The spin histories of the AMXPs are valuable because of what they reveal about the properties of the star and the different torque mechanisms that may operate. In terms of stellar properties we are interested in the moments of inertia of, and degree of coupling between, the various components - particularly the solid crust and superfluid core [16]. Possible angular momentum sources and sinks include material/magnetic torques as matter from the companion accretes via a magnetically-threaded accretion disk [22], magnetic dipole radiation, jets [28], and gravitational wave emission (with various different mechanisms capable of generating a quadrupole [18]). The challenges inherent in this type of analysis and modeling are well illustrated by earlier studies of the high magnetic field (slower-rotating) X-ray pulsars [29]. In the AMXPs magnetic fields are weaker, but gravitational wave torques could play a larger role since gravitational wave emission scales strongly with rotation rate.

Spin histories for the AMXPs are constructed using standard radio pulsar timing techniques that involve measuring phase shifts between folded pulse profiles. Pulse profiles in the AMXPs are however notoriously variable, since they are affected by fluctuations in the accretion flow. This additional noise leads to pulse phase wander which is sufficiently large that it may mimic or mask genuine spin variation. As evidenced by the lively discussions on this topic at the Amsterdam workshop, the level of confidence in inferred values of spin derivatives remains a matter of vigorous debate [30, 31, 32, 33, 34, 35, 36].

Timing using burst oscillations could in principle provide an independent test of spin variation, since the emission mechanism for the nuclear-powered pulsations is thought to be quite different to the accretion-powered pulsations. Drifts in burst oscillation frequency, of course, complicate this task (making analysis for SAX J1808.4-3658 very difficult). However XTE J1814-338, which has exceptionally stable burst oscillations and a large burst sample across its one recorded 50 day outburst, is a highly promising source for nuclear-powered pulsation timing. Its accretion-powered pulsations show substantial phase wander, some of which could be due to changes in spin frequency [33]. A full timing analysis of the burst oscillations has now been completed [37], and the results have proved surprising. The nuclear-powered pulsations are completely phase-locked (with zero offset) to the accretion-powered pulsations, tracking perfectly all of the phase wander throughout the main part of the outburst. At first glance, this does little to

resolve the debate, since the result is certainly consistent with some or all of the changes being caused by spin variation. What it does do, though, is to provide a major constraint on models that attempt explain the phase wander without requiring spin changes. Any model must now be able to explain locked jitter in two totally different types of pulsation. There are mechanisms that could in principle do this (see Section 4 and the discussion in [37]), but these need to be tested further. If they do not prove viable, this would support there being at least some genuine spin variation.

## 4. BURST OSCILLATION MECHANISMS

The nature of the brightness asymmetry that causes nuclear-powered pulsations is still not understood for any source, AMP or otherwise. The status of the various different models may be summarized as follows:

- **Hotspot spread:** A localized growing hotspot is expected to exist in the burst rise, since ignition should not start simultaneously across the stellar surface [38]. There is evidence that expanding hot spots are related to the presence of burst oscillations in the rising phases of some bursts (see for example [39, 40]). However only a very small percentage of the burst sample has been subject to this rigorous level of analysis, and it remains to be seen whether the entire sample is consistent with this model. Where the hotspot model runs into real difficulty is in the burst tail. Once the flame front has spread across the star, the remaining asymmetry should not be strong enough to explain the continued presence of pulsations [4].
- **Thermonuclear hurricanes:** The Coriolis force can act to confine the burning area during the burst rise [41] (making this model to some degree a variant of the spreading hotspot). This may explain the presence of oscillations in the burst rise, although like the previous model it has yet to be put to rigorous test. The authors of this study [41] conjectured that the resulting unstable flows might lead to the development of similar localized phenomena in the burst tail. Subsequent simulations have apparently not borne this out, although magnetic effects during vortex formation remain to be investigated fully (contribution by Levin, this workshop).
- **Surface modes:** Global oscillations may be excited by flame spread, generating a brightness asymmetry that could persist throughout the burst tail. Attention has focused on buoyant r-modes in the neutron star ocean, since these have frequencies close to the stellar spin rate [42]. The mode model also provides a natural explanation for frequency drift seen in the tails of many bursts, since the frequency will change as the surface layers cool. Unfortunately the model over-predicts the observed drifts: coupling to a crustal interface wave, which was proposed as a way of limiting the drift [43], has now been shown to be inefficient [44]. Alternative mode types including photospheric [42] and shearing oscillations [45] also have shortcomings [44], but magnetic effects may play a significant role in determining mode behaviour (contribution by Cumming, this workshop).

The most significant contribution that the AMPs have made to our understanding of the burst oscillation mechanism is of course the requirement that the frequency should lie within a few Hz of the spin rate: all of the models listed above take this as a basic

premise. However these sources are also valuable in other ways. They are the only systems in which we can reliably gauge the role of the magnetic field in generating and maintaining burst oscillations. They also let us assess the influence of asymmetric fuel deposition. For the weak magnetic fields of the AMXPs fuel should spread before reaching ignition depth [46] but there are other local effects at the deposition point, such as higher temperature, that may be important.

These factors have motivated efforts to measure the properties of the burst oscillations of the AMPs and compare them to both the accretion-powered pulsations and the other burst oscillation sources. Key results, drawn from many different references (non-AMPs [12, 47, 48, 49, 5]; intermittent AMPs [50, 14, 11]; persistent AMPs [7, 8, 9, 51, 37, 52, 53]) are as follows:

1. For most NMPs, including Aql X-1, burst oscillations are only detectable in some bursts. They are more prevalent at high accretion rates and in short, He-rich bursts. SAX J1808.4-3658 and XTE J1814-338 show them in every burst despite low accretion rates and, for XTE J1814-338, long mixed H/He bursts.
2. The amplitudes of burst oscillations in the non-AMPs lie in the range 2-20% RMS. The absolute amplitudes of the burst oscillations from the AMPs are similar, but never exceed the accretion-powered pulsation amplitudes.
3. SAX J1808.4-3658 and XTE J1814-338 are the only sources where the burst oscillations have detectable harmonic content, albeit at a lower amplitude than in the accretion-powered pulsations.
4. The burst oscillations for most NMPs, and Aql X-1, have an amplitude that rises with energy. For XTE J1814-338, and most of the bursts from SAX J1808.4-3658, amplitude falls with energy. This behaviour is inconsistent with both simple hotspot and surface mode models [54]. A fall in amplitude with energy is also seen in the accretion-powered pulsations of all of the persistent AMPs, but it is not clear that the same mechanism could explain this effect in both pulsation types.
5. Burst oscillations from the AMPs show no detectable phase lags. This behaviour is similar to that of the other NMPs, which show at most marginal hard lags.
6. The frequency drifts seen in Aql X-1 are typical of the other NMPs: a slow rise during the burst to a saturation frequency. The two persistent AMPs with burst oscillations are very different. For XTE J1814-338 there are no detectable drifts except in the final brightest burst, which has a small drop in frequency in the burst rise. For SAX J1808.4-3658 burst oscillation frequency rises rapidly by several Hz in the rising phase of the bright bursts, in some cases overshooting the spin frequency. Drifts in the burst tails, however, are minimal.
7. The nuclear-powered pulsations in XTE J1814-338 are completely phase-locked to the accretion-powered pulsations even though the latter show substantial phase wander over the course of the outburst.

So what do these results tell us about the burst oscillation mechanism? Firstly, that existing models are no better at explaining AMP burst oscillation properties than they are at explaining the rest of the NMP population. The fall of amplitude with energy, and frequency overshoot, for example, are not predicted by any current model.

The second important question is whether we are looking at a continuum of behaviour that could be attributed to one mechanism (with differences being set by varying magnetic field strength, for example). The burst oscillation properties of Aql X-1 sit comfortably within the general population, perhaps not surprisingly for a source that is a very intermittent AMP. The non-detection of burst oscillations from the other two intermittent AMPs is also not too worrying, since many LMXBs with bursts fail to show burst oscillations. SAX J1808.4-3658 is more of a challenge. Whether this source is consistent with a unified model seems to depend primarily on the mechanism responsible for the frequency shift. The source could fit if, as suggested by [7], the rapidity and magnitude of the frequency shift could be shown to depend strongly on magnetic field strength (or the degree of misalignment between the magnetic field and rotational pole).

The remaining source, XTE J1814-338, is an oddball. The properties of its nuclear-powered pulsations differ in almost every way from the rest of the sample. The phase-locking of the two types of pulsation, however, suggests that the presence of nuclear-powered pulsations in this source may be related to premature ignition and subsequent stalling of the flame front [37]. This is a rather exciting possibility, but is sadly unlikely to explain the presence of oscillations in the bright, He-rich bursts of the other sources (flame front stalling being less likely in such bursts). This leaves open the intriguing possibility that there may be at least two different burst oscillation mechanisms.

## 5. CONCLUSIONS

The discovery of nuclear-powered pulsations from the AMPs has cemented the link between burst oscillation frequency and spin frequency. In addition to confirming the absence of rapid rotators (now a major problem for evolutionary models), this has imposed the strongest single constraint on candidate burst oscillation mechanisms. Despite this huge clue, the mechanism remains elusive: but continuing analysis of the AMPs is providing tantalising evidence that is driving the development of new theoretical models.

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